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New Law for Crack Propagation in Solid Propellant Material

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Determination of the relationships that govern crack propagation in propellant material is essential for predicting the failure of grains. Until now, the most useful crack propagation law was the Paris one which gives a power relation between the stress intensity factor and the crack propagation rate. However, this relation appears to be inadequate for a correct representation of crack propagation in viscoelastic materials; by using the linear cumulative damage theory and the concept of failure area, a modified law may be chosen: $a_T\,\mathrm{d}a/\mathrm{d}t = A/t_m$. This proposed law allows a further reduction of the dispersion of the results for tensile experiments on uniaxial precracked specimens. The propellants used in this study are polyurethane and carboxyterminated polybutadiene propellants.

Nomenclature

A, A'	= constant (crack propagation)
a	= crack length
a_T	= W.L.F. shift factor
b	= half of the width of the specimen
D	= damage
$\mathrm{d}a/\mathrm{d}t$	= crack velocity
E_I	= constant
E_{rel}	= relaxation modulus
k	= constant (damage law)
K_I	= stress-intensity factor
N	= number of cycles in a fatigue experiment
n	= constant (power law)
p	= constant (damage law)
q	= constant (crack propagation)
R	= strain rate
T	= temperature
t	= time
t_α	= characteristic time
t_m	= time corresponding to the maximum stress σ_m during a uniaxial tensile experiment
α	= length of failure zone
ψ	= geometric factor
σ	= applied stress
σ_0	= constant (power law)
σ_m	= maximum stress during a uniaxial tensile experiment
Γ	= fracture energy

Introduction

A MODERN approach to the failure behavior of solid propellants can be based on crack propagation analysis. This method can be very useful in the study of the mechanical integrity of grains submitted to variable loading conditions (mainly storage and firing).[1]

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Paris conventional theory[2] for fatigue failure of elastic materials is characterized by the use of the stress intensity factor increment and by a power law dependency on the rate of crack propagation:

$$\mathrm{d}a/\mathrm{d}N = A\,[\Delta K]^{q} \tag{1}$$

A generalization of the above-mentioned theory to solid propellant was proposed by Schaeffer[3]:

$$a_T\mathrm{d}a/\mathrm{d}t = AK_I^q \tag{2}$$

and a numerical application for a CTPB propellant gave the formula

$$a_T\mathrm{d}a/\mathrm{d}t = 100\,K_I^6 \tag{3}$$

the units being bar-cm$^{1/2}$ for K_I and cm S^{-1} for $\mathrm{d}a/\mathrm{d}t$; a_T is the W.L.F. (Williams-Landel-Ferry)[4] shift factor with $a_T = 1$ for a temperature of 20°C. A development along conventional fracture mechanics and viscoelastic characterization lines by R. Schapery[5-7] and experimental applications by Francis[8] and by S.R. Swanson[9] lead to the same kind of law [Eq. (2)].

However, it was observed by Beckwith and Wang[10] that the value $q = 2$ gives a much better explanation of experimental results; recently, they proposed that factor A should be dependent on the strain level ϵ in the specimen in order to obtain the different curves

$$a_T\mathrm{d}a/\mathrm{d}t = A(\epsilon)K^6 \tag{4}$$

by vertical translations for different values of ϵ (Fig. 1). It must be pointed out that this work is the only material to date which gives the value $q = 2$ (the propellant used was a CMDB propellant).

In this paper, another type of explanation is proposed, for possible values of this parameter. A theoretical development based on the linear cumulative damage theory and the concept of failure is given; application of the theory to CTPB and PU propellants shows a substantial reduction of the dispersion of experimental results.

Theoretical Developments

A. Intuitive Approach

Begin with an intuitive approach using linear viscoelasticity; find a normalizing factor $f(\sigma_m, t_m)$ applied to Eq. (2) in the form

$$a_T\mathrm{d}a/\mathrm{d}t = AK_I^q/f(\sigma_m, t_m) \tag{5}$$

so as to explain conventional time-dependent failure of a uniaxial specimen:

$$\sigma_m^q t_m = \text{constant} \tag{6}$$

where σ_m is the maximum stress for a uniaxial constant-rate tensile experiment and t_m is the corresponding time. Using a power law for the relaxation modulus

$$E = E_1 t^{-n} \tag{7}$$

and R being the rate of the tensile experiment, the far-field stress can be calculated in the form

$$\sigma = \int_0^t E(t-\tau) R \mathrm{d}t = \frac{RE_1}{1-n} t^{1-n} = \sigma_0 t^{1-n} \tag{8}$$

For the stress intensity factor, the conventional expression

$$K_I = \psi \sigma (\pi a)^{1/2} \tag{9}$$

is used, where a is the depth of the crack and ψ a shape factor.

Now integrate Eq. (5), using Eqs. (8) and (9), assuming that the dimension of the precrack a_0 is the same as that of the perchlorate particle and that 2b is the width of the specimen.

$$\int_a^{2b} \frac{a_T}{a^{q/2}} \mathrm{d}a = \int_0^{t_m} \frac{A(\psi\sigma_0\sqrt{\pi})^q t^{q(1-n)}}{f(\sigma_m, t_m)} \mathrm{d}t$$

$$= \frac{A(\psi\sqrt{\pi})^q (\sigma_0 t_m^{1-n})^q t_m}{f(\sigma_m, t_m)} \tag{10}$$

so that $\sigma_m^q t_m / f(\sigma_m, t_m) = \text{constant}$, which according to Eq. (5) leads to the formula

$$a_T \mathrm{d}a / \mathrm{d}(t/t_m) = A'(K_I/\sigma_m)^q \tag{11}$$

Equation (11) corresponds to a normalization of Eq. (2) in σ and t.

B. Analytical Approach

Irwin[11] showed that for an infinite elastic sheet, the analytical form of the stress can be written

$$\sigma(x) = \frac{K_I}{\sqrt{2\pi(x-a)}} \tag{12}$$

x being the abscissa of the local point and a the abscissa of the crack-tip (for a single-edged notch specimen, it should be an analogous form with a different constant term). Then, this author assumed the existence of a plastic zone, the length of which is α and where the maximum stress σ_m is reached; to maintain the equilibrium of forces, he had to suppose the existence of a fictitious crack of length $a + \alpha/2$ (Fig. 2).

A similar development is noted by Schapery[5-7] in terms of the crack-tip stresses and of the Barenblatt model to account for the damage zone given by α. Presently, the restrictions imposed on Schapery's analytical developments are neglected.

Equation (12) leads to

$$\alpha = K_I^2 / \pi \sigma_m^2 \tag{13}$$

Let us introduce the damage $D(t)$ according to the linear theory proposed by Miner[12] and developed by Farris and Falabella[13,14] for polymeric materials:

$$D(t) = \frac{1}{k} \int_0^t \sigma(t)^p \mathrm{d}t \tag{14}$$

at failure $D(t_m) = 1$ so that

$$k = \int_0^{t_m} \sigma(t)^p \mathrm{d}t \tag{15}$$

using Eq. (8) for the far-field stresses

$$D(t) = \frac{1}{k} \sigma_0^p \int_0^t t^{p(1-n)} \mathrm{d}t = \frac{\sigma^p t}{k[p(1-n)+1]} \tag{16}$$

and

$$D(t_m) = 1 = \sigma_m^p t_m / k[p(1-n)+1] \tag{17}$$

The rate of crack propagation is calculated in the following manner: using the concept of cumulative damage at the point of abscissa x

$$D(x,t) = \frac{1}{k} \int_0^t \sigma^p(x,\tau) \mathrm{d}t \tag{18}$$

and considering the time t_r when $a = a_r$ and $\alpha = \alpha_r$, then $\sigma(x) = \sigma_m$ for $a_r < x < a_r + \alpha_r$ and $D(a_r, t_r) = 1$. If t_l is the time when $\sigma(x)$ reaches σ_m and $t_r - t_l$ is the period of time necessary for the crack to propagate from $a_r - \alpha_r$ to a_r, then the following formula is obtained:

$$\frac{1}{k} \int_0^{t_r} \sigma^p(x,\tau) \frac{\mathrm{d}\tau}{a_T}$$

$$= \frac{1}{k} \int_0^{t_l} \sigma^p(x,\tau) \frac{\mathrm{d}\tau}{a_T} + \frac{1}{k} \int_{t_l}^{t_r} \sigma^p(x,\tau) \frac{\mathrm{d}\tau}{a_T} = 1 \tag{19}$$

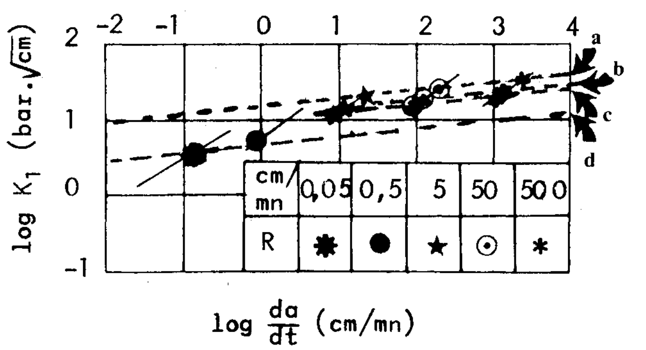

Fig. 1 Effect of strain level on crack propagation in CMDB propellant (from Ref. 10): ϵ = 22%, a; 16%, b; 13.5%, c; 7.5%, d.

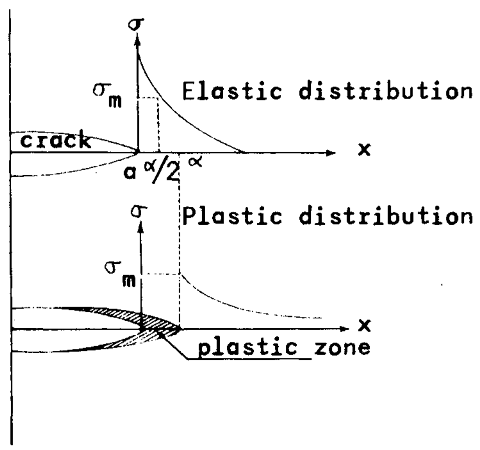

Fig. 2 Stress distribution near the crack tip.

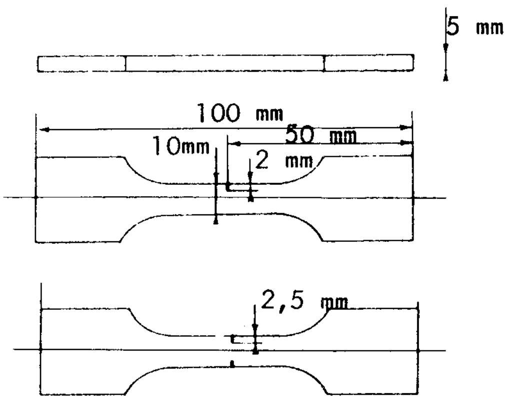

Fig. 3 Single- and double-edged notch specimen.

$t_r - t_l$ being small, the rate of propagation da/dt can be considered as a constant during the propagation, so that $da/dt = \alpha/(t_r - t_l)$ and Eq. (19) becomes, after integration,

$$\int_0^{t_l} \sigma^p(x,\tau)\frac{d\tau}{a_T} + \frac{\sigma_m^p \alpha}{a_T da/dt} = k \qquad (20)$$

Then assume that $\int_0^{t_l} \sigma^p(x,\tau) d\tau / a_T$

is much smaller than $\sigma_m^p \alpha/(a_T\ da/dt)$ (that is due to the large value of p and can be proved by numerical applications); from $\sigma_m^p \alpha/(a_T\ da/dt) = k$ and using Eq. (17), the following relation is obtained:

$$a_T\,da/dt = [p(1-n)+1]\alpha/t_m \qquad (21)$$

Equation (21) can be compared to the expression obtained by Knauss[15] and Schapery[5-7]: $da/dt = \alpha/3\ t_\alpha$. This shows that the characteristic time t_α they introduced must be related to the time t_m at maximum stress. Finally Eqs. (13) and (21) make it possible to obtain

$$a_T\,da/dt = [p(1-n)+1]K_I^2/\pi\sigma_m^2 t_m \qquad (22)$$

Experimental Background and Applications

Except in one case which is mentioned hereafter, tensile experiments were conducted on single-edged notch specimens with a precrack 2 mm in length (Fig. 3). The crack was initiated at a known depth with a razor blade, then propagated to get a natural aspect.

Use the analytical development[16] for K_I:

$$K_I = [1.99 - 0.41(a/2b) + 18.70(a/2b)^2 - 38.48(a/2b)^3]\sigma\sqrt{a} \qquad (23)$$

where a is the actual length of the crack, determined by photography during the test, and σ is the far-field stress.

Three sets of uniaxial tensile experiments were conducted:

1) Polyurethane propellant A1 (PUA1) was tested at different temperatures (40°C, 20°C, 10°C, −10°C) and, for each temperature, at three tensile rates (50, 5, 0.5 mm/min). Figure 4 describes the results in the space combining reduced rate of crack propagation $a_T\,da/dt$ and stress intensity factor K_I. The W.L.F. shift factor a_T was obtained from relaxation experiments.

2) Polyurethane propellant A2 (PUA2) was tested at different temperatures (60°C, 20°C, 0°C, −40°C) at a tensile rate of 5 mm/min, except at 20° C, where three different rates were used (0.5, 5.0, 890 mm/min); at the temperature of 20°C and rate of 0.5 mm/min, a double-edged notch specimen was

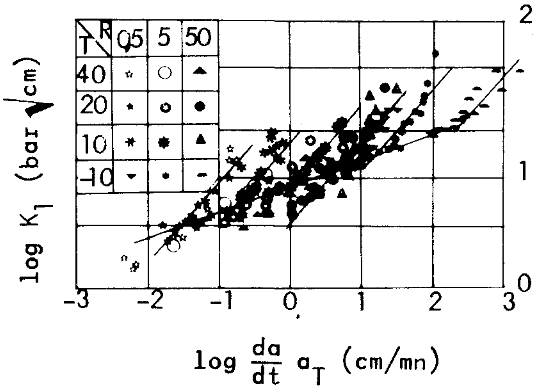

Fig. 4 Paris law for PUA1 propellant.

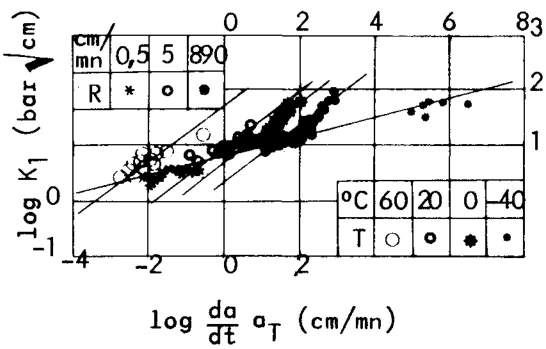

Fig. 5 Paris law for PUA2-B propellant.

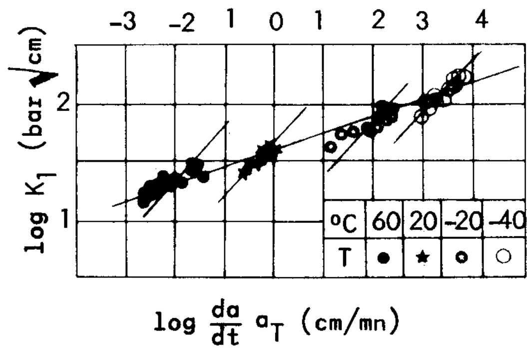

Fig. 6 Paris law for CTPB-B propellant.

tested (Fig. 3); the analytical development used for K_I is[17]

$$K_I = (2/\sqrt{\pi})\sigma\sqrt{c} \qquad (24)$$

c being half of the distance between the tips of the cracks.

3) Carboxy-terminated polybutadiene propellant (CTPB-B) was tested at different temperatures (60° C, 20° C, −20° C −40° C) at a tensile rate of 5 mm/min.

From the experimental results (Figs. 4-6), it seems that there are two ways of explaining the data: 1) using the global law, Eq. (2), with $q = 6$ (the conventional approach, corresponding to dashed lines on the figures), or 2) using the same form, Eq. (2), but with $q = 2$ (and different values of constant A, corresponding to solid lines on the figures).

Using the relation of Eq. (22) to explain the data and eliminating t_m from Eqs. (22) and (17) leads to

$$a_T\ da/dt = K_I^2 \sigma_m^{p-2}/k\pi \qquad (25)$$

k and p determined from tensile experiments on an undamaged specimen. In Figs. 7-9, log $K_I^2\ \sigma_m^{p-2}$ is plotted vs a_T

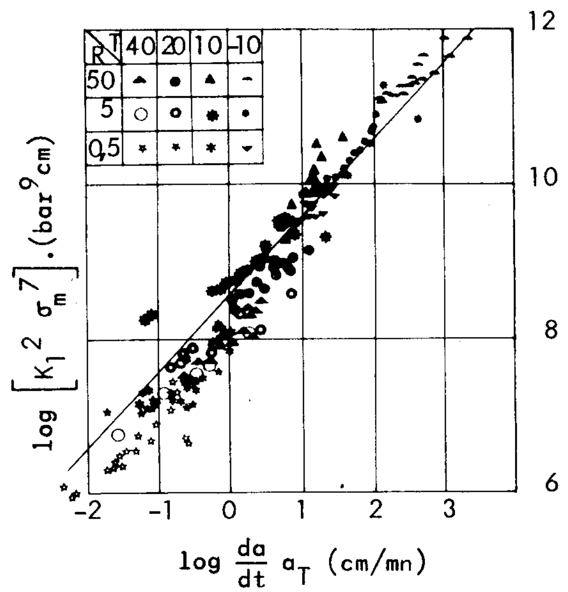

Fig. 7 Modified crack growth law for PUA1 propellant.

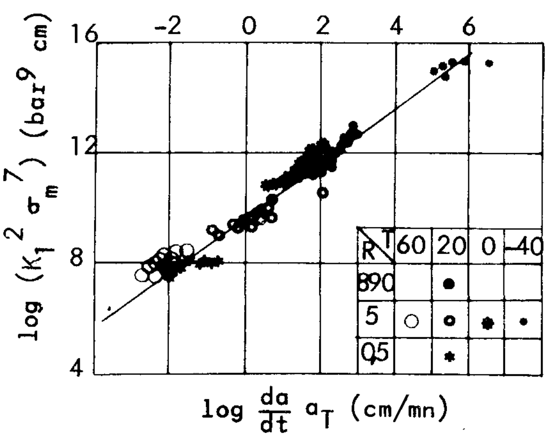

Fig. 8 Modified crack growth law for PUA2 propellant.

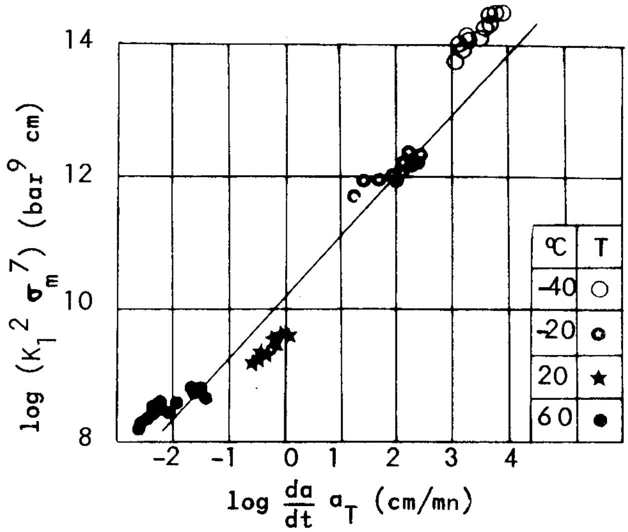

Fig. 9 Modified crack growth law for CTPB-B propellant.

da/dt. A close similarity is found between these experimental results and the straight line with slope 1. The ratio of the value of k obtained from theory and from experimental data is not superior to 3, which gives very close values.

Conclusion

The new proposed law which has been verified for constant-rate tensile tests leads to a much better explanation of experimental results of crack propagation than the Paris conventional law. It also accounts for the data better than the law given by Schapery[5-7] or Beckwith and Wang,[10] especially in view of the nonlinear flexibility of the Schapery and Knauss developments. This study should be carried on in fields such as creep or simultaneous tensile and cooling loads; then verifications should be carried out on actual propellant grains (either subscale or full-scale motors as a final proof).

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